# Section 1.2: Basic Classes of Functions

We have studied the general characteristics of functions, so now let’s examine some specific classes of functions. We begin by reviewing the basic properties of linear and quadratic functions, and then generalize to include higher-degree polynomials.

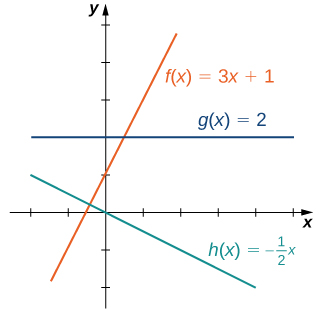
## Linear Functions and Slope

One of the distinguishing features of a line is its slope. The slope is the change in for each unit change in . The slope measures both the steepness and the direction of a line.

Consider the line passing through points and . Let and denote the changes in and , respectively. The **slope** of the line is

.

**Linear functions** have the form , where and are constants. If , the graph of the line rises as increases (i.e. is increasing on . If , the graph of the line rises as decreases (i.e. is decreasing on . If , the line is horizontal.



Consider a line passing through the point with slope . The equation

Is the **point-slope equation** for that line.

Consider a line with slope and -intercept . The equation

Is an equation for that line in **slope-intercept form**.

The **standard form** of a line is given by the equation

,

Where and are both not zero. This form is more general because it allows for a vertical line, .

Examples

1. Find the slope of the line that passes through and and indicate whether the line is increasing, decreasing, horizontal, or vertical.
2. Consider the line passing through the points and ,
   1. Find the slope of the line.
   2. Find an equation for this linear function in point-slope form.
   3. Find an equation for this linear function in slope-intercept form.
3. Jessica leaves her house at 5:50 am and goes for a 9-mile run. She returns to her house at 7:08 am. Answer the following questions, assuming Jessica runs at a constant pace.
   1. Describe the distance (in miles) Jessica runs as a linear function of her run time (in minutes).
   2. Sketch a graph of .
   3. Interpret the meaning of the slope.

## Polynomials

A linear function is a special type of a more general class of functions: polynomials. A **polynomial function** is any function that can be written in the form

For some integer and constants , where . The value is called the **degree** of the polynomial; the constant is called the **leading coefficient**.

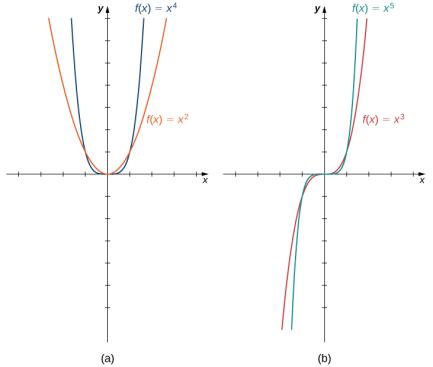
### Power Functions

Some polynomial functions are power functions. A **power function** is any function of the form , where and are any real numbers. The exponent in a power function can be any real number, but here we consider the case when the exponent is a positive integer.

If the exponent is a positive integer, then is a polynomial.

If is even, then is an even function because if is even.

If is odd, then is an odd function because if is odd.



### Behavior at Infinity

To determine the behavior of a function as the inputs approach infinity, we look at the values as the inputs, , become larger. For some functions, the values of approach a finite number. For other functions, the values may not approach a finite number but instead may become larger for all values of as they get larger.

### Zeros of Polynomial Functions

Another characteristic of the graph of a polynomial function is where it intersects the -axis. The determine where a function intersects the -axis, we need to solve the equation for . In the case of a quadratic function, finding the -intercept(s) requires finding the zeros of a quadratic equation: . In some cases, it is easy to factor the polynomial to find zeros. If not, we make use of the quadratic formula.

Consider the quadratic equation

Where . The solutions of this equation are given by the quadratic formula

.

If the discriminant , this formula tells us there are two real numbers that satisfy the quadratic equation. If , this formula tells us there is only one solution, and it is a real number. If , no real numbers satisfy the quadratic equation.

Examples: For the following functions, describe the behavior of as , find all zeros of , and sketch a graph of .

### Mathematical Models

A large variety of real-world situation can be described using mathematical models. A mathematical model is a method of simulating real-life situations with mathematical equations.

Example

A company is interested in predicting the amount of revenue it will receive depending on the price it charges for a particular item. Using their data, the company arrives at the following quadratic function to model the revenue (in thousands of dollars) as a function of price per item :

for .

1. Predict the revenue if the company sells the item at a price of and .
2. Find the zeros of this function and interpret the meaning of the zeros.
3. Sketch a graph of .
4. Use the graph to determine the value of that maximizes revenue. Find the maximum revenue.

## Algebraic Functions

By allowing for quotients and fractional powers in polynomial functions, we create a larger class of functions. An **algebraic function** is one that involves addition, subtraction, multiplication, division, rational powers and roots. Two types of algebraic functions are rational functions and root functions.

A **rational function** is any function of the form , where and are polynomials.

A **root function** is a power function of the form , where is a positive integer greater than one.

Example: For each of the following functions, find the domain and range.

## Transcendental Functions

Thus far, we have discussed algebraic functions. Some functions, however, cannot be described by basic algebraic operations. These functions are known as transcendental functions because they are said to “transcend,” or go beyond, algebra. The most common transcendental functions are trigonometric, exponential, and logarithmic functions.

Example: Classify each of the following functions as algebraic or transcendental.

## Piecewise-Defined Functions

Sometimes a function is defined by different formulas on different parts of its domain. A function with this property is known as a **piecewise-defined function.**

Examples

1. Sketch a graph of the following piecewise-defined function:
2. In a big city, drivers are charged variable rates for parking in a parking garage. They are charged $10 for the first hour or any part of the first hour and an additional $2 for each hour or part thereof up to a maximum of $30 for the day. The parking garage is open from 6 am to 12 midnight.
   1. Write a piecewise-defined function that describes that cost to park in the parking garage as a function of hours parked .
   2. Sketch a graph of this function .

## Transformations of Functions

We have seen several cases in which we have added, subtracted, or multiplied constants to form variations of simple functions. These types of variations are types of **transformations of a function.**

We can summarize the different transformations and their related effects on the graph of a function below:

**Transformation of**

: Vertical shift up units

: Vertical shift down units

: Shift left units

: Shift right units

: Vertical stretch if ; Vertical compression if

: Horizontal stretch if ; Horizontal compression if

: Reflection about the -axis

: Reflection about the -axis

If the graph of a function consists of more than one transformation of another graph, it is important to transform the graph in the correct order.

Given a function , the graph of the related function can be obtained from the graph of by performing the transformations in the following order:

1. Horizontal shift of the graph of . If , shift left. If , shift right.
2. Horizontal scaling of the graph of by a factor of . If , reflect the graph about the -axis.
3. Vertical scaling of the graph of by a factor of . If , reflect the graph about the -axis.
4. Vertical shift of the graph of . If , shift up. If , shift down.

Examples: For each of the following functions, sketch a graph by using a sequence of transformations of a well-known function.